ONDERZOEKSRAPPORT NR. 8513

AIRCRAFT LINE AND LIGHT MAINTENANCE MANPOWER ALLOCATION AND SCHEDULING

BY

WILLY R. BUYSSE
WILLY S. HERROELEN

D/1985/2376/18

AIRCRAFT LINE AND LIGHT MAINTENANCE MANPOWER ALLOCATION AND SCHEDULING

by

Willy R. BUYSSE*

Willy S. HERROELEN**

* Production Manager, Assistant V.P. Maintenance, SABENA.
This research is an outgrowth of a thesis submitted by this author as partial fulfillment of the requirements imposed by the MBA program at KU. Leuven.

** Department of Applied Economic Sciences, KU. Leuven.

The authors are very much indebted to Prof. W. Gochet for his comments made on an earlier draft of this paper.

AIRCRAFT LINE AND LIGHT MAINTENANCE MANPOWER ALLOCATION AND SCHEDULING

ABSTRACT

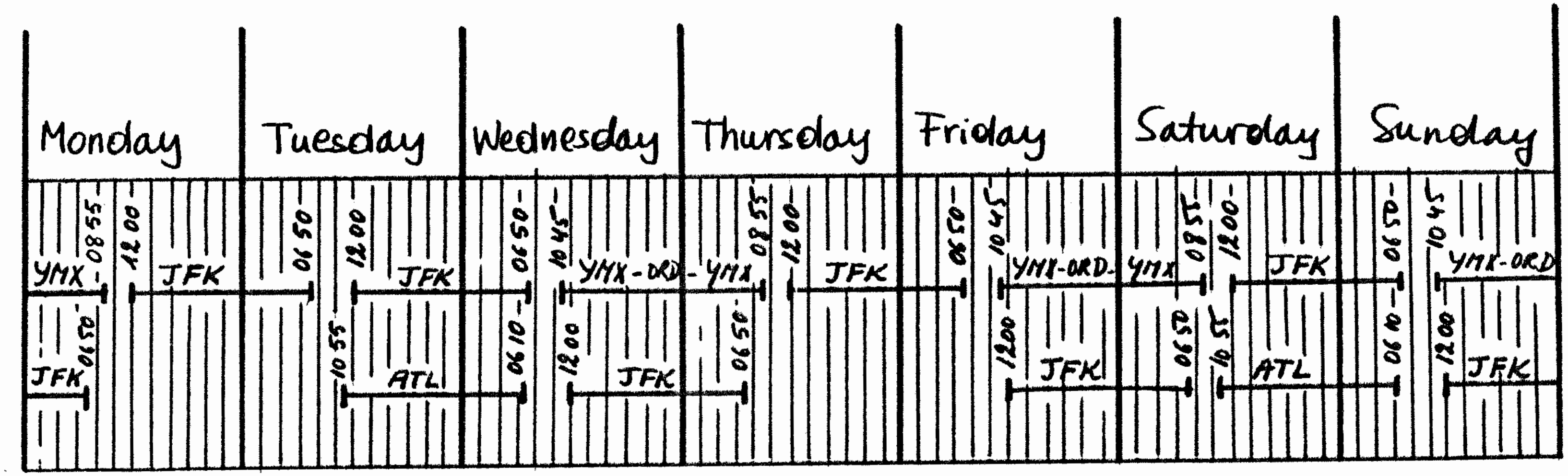
The present paper presents an integrated solution procedure for the scheduling of crews for line and light maintenance for a fleet of aircraft operated on medium and long range transcontinental routes. Coping with the stochastic nature of both the aircraft arrival times and the manhour requirements for various maintenance checks, and given a proper choice of the service level required, the minimum line and light maintenance manpower requirements are determined for each shift. With the objective of minimizing overall maintenance costs, integer linear programming models are built to decide on the optimal number of maintenance crews to be used on each of the allowable work and recreation schedules satisfying maintenance manpower requirements and constraints resulting from labor contract specifications.

A method is described for sequencing the work and recreation schedules in order to keep workers' aversion to a minimum. The paper includes the discussion of numerical results derived from a practical case study involving a fleet of two jumbo aircraft operated by an international Belgian airline.

I. INTRODUCTION

Aircraft maintenance and overhaul costs represent about 10 % of the unit operating cost of an air carrier [1]. For the last eight years they rank fifth in the nine cost categories defined by IATA. Both maintenance and overhaul are very labour intensive activities, requiring skilled workers and employees. Almost three quarters of the budget of a typical airline maintenance department goes to direct labour. Careful determination of maintenance manpower requirements and close matching of manpower requirements and availabilities may lead to serious improvements in unit operating costs. The present paper is concerned with the direct airframe labour costs for line and light maintenance incurred in the mainbase of carriers operating fixed flight schedules.

The flight routes for a certain aircraft type are determined by the commercial department of an airline in a flight schedule. This flight schedule has a weekly cycle and is valid for one season. One program is defined for the summer season (basically from April 1 till October 31), another for the winter season (from November 1 till March 31). Figure 1 gives the winter 84/85 flight schedule for the fleet of two B747 aircraft which will make up the case study for this paper. The arrival and departure times specified by the flight schedule are published in the airline's time table and are called respectively scheduled time of arrival (STA) and scheduled time of departure (STD). The difference between STD and STA is called the scheduled turnaround time (STT). If this turnaround time exeeds a certain value defined by the airport authorities, the aircraft will be towed away from the boarding gate and the turnaround time will be called scheduled grounding time (SGT). Actual arrival times almost never equal the scheduled values. Consequently, the actual arrival times will be described by a proper density function which for a particular route will be different for the winter and summer season but which will not differ from day to day within a season. Timely departures on the other hand are very important to airline passengers. Departure times must be guaranteed at all times and will be



Legend : - arrival and departure times are given in Greenwich mean time (GMT)
- flight destinations are indicated by an international three letter code (e.g. JFK is Kennedy Airport, New York)
- bold lines indicate the time periods during which the aircraft is not in its mainbase (Brussels Airport)

Figure 1 : Winter 84/85 flight schedule example for two B747 aircraft.

considered as deterministic due dates. As a result of all this, the actual turnaround times and the actual grounding times are not known deterministically.

All scheduled maintenance and inspection checks to be performed at specified intervals are fully described in a standard maintenance schedule which is issued for each type of aircraft. In this paper we only deal with the so-called line maintenance and light maintenance checks to be performed at the aircraft's mainbase. Line maintenance checks may be performed at the boarding gate during the turnaround time - the so-called ramp maintenance - or during the scheduled grounding time in the maintenance hangar. The majority of light maintenance checks must be performed in the maintenance hangar. The resource profiles (worker qualification, tooling, instruments, etc.) required to perform the maintenance and inspection tasks as defined by the maintenance schedule are known for each check. However, during the performance of the specific maintenance tasks and inspections themselves, certain irregularities may show up. The quantity of these maintenance discrepancies and the manpower required to fix them are not known in advance. In addition the flight crew of the aircraft may report certain irregularities during the flight which will also give rise to a maintenance work package which is not known in advance. The overall manhour requirement for fixing flight and maintenance discrepancies will be described by a proper density function.

The basic purpose of this paper is to develop an integrated solution procedure for the scheduling of line and light maintenance crews for a fleet of two B747 aircraft operated on medium and long range transcontinental routes according to flight schedules as given in Figure 1. Coping with the stochastic nature of both the aircraft arrival times and the manhour requirements for the various maintenance tasks described above, we will derive an overall manhour requirement density function f(mh) and an actual time of arrival density function f(ATA) according to the rules described in the next section. Given the deterministically scheduled departure times (STD) and a proper choice of the service level, SL, defined as the probability that a certain flight will depart on schedule, we will show in sufficient detail how to determine the minimum line and light maintenance manpower requirements, m_{SL}^{s}, for each given shift s.

With the objective of minimizing overall maintenance costs an integer linear programming model will be set up in Section III to decide on the optimal number of maintenance crews to be used on each of the allowable work and recreation schedules satisfying the various maintenance manpower requirements and specific constraints resulting from labor contract specifications.

As will be shown in Section IV, the weekly work and recreation schedules can be sequenced within larger rotating shift schedule cycles in many different ways. A method will be described for sequencing the allowable work and recreation schedules in order to keep workers' aversion to a strict minimum. It will become apparent from our subsequent discussion that the integrated solution procedure developed in this paper, spans several stages of the five-stage manpower scheduling framework developed by Tien and Kamiyama [7], who also provide an excellent overview of the manpower scheduling literature.

II. DETERMINING MAINTENANCE MANPOWER REQUIREMENTS

The overall procedure for determining the maintenance manpower requirements relies on the derivation of proper density functions for the aircraft arrival times and the overall maintenance manhour requirements.

1. Arrival time density functions

Aircraft arrival times are influenced by a number of factors such as delays caused at departure and stopover stations, atmospheric conditions (jet streams, storms) and so on. As a result aircraft arrival times are stochastic in nature. Past seasonal arrival time data may be used to estimate a proper actual arrival time density function f(ATA). Data available at the airline under study, however, refer to the arrival times of flights connecting two stations along the same route of the flight schedule but made on different days of the week during three winter seasons (1981/82, 1982/83 and 1983/84) and two summer seasons (1982 and 1983). This calls for a statistical test to determine whether different arrival time density functions should be used for the different days of the week and from one season to the other.

A k-sample analogue of the Kolmogorov-Smirnov test developed by Kiefer [3] was adapted for this purpose. Basically Kiefer tests the so-called homogeneity hypothesis $H_0 : F_1 = F_2 = \dots F_k$ where the F_j, $1 \leq j \leq k$ denote the unknown continuous arrival time distributions against the alternative hypothesis H_1 that there exists a set $\{F_1, \dots, F_j\}$ that violates H_0. Kiefer's test has the interesting feature that it does not require the tested distribution functions to be of a specific type (such as the normality condition imposed by a two-way ANOVA) nor that all distribution functions need to be of the same form with possible shifts allowed (such as imposed by the Kruskal-Wallis multisample location test [4,5]). In addition the test satisfies the requirement imposed by the available data that sample sizes are not necessarily equal. The various adaptations made to Kiefer's test procedure, mainly caused by the lengthy computations needed to perform Kiefer's procedure on our 35 data samples (a daily flight during 3 winter and 2 summer seasons), are beyond the scope of this paper and are fully described in [1]. The procedure indicated actual aircraft arrival times to be independent of the particular day of the week. In addition, the test indicated that the data for 398 flights during the past winter seasons and 384 flights during the past summer seasons could be pooled to estimate the winter and summer season arrival time density functions for the New York - Brussels flight, that three data sets (58, 42 and 39 flights) could be used to estimate three winter season arrival time density functions and that a pooled summer season data set (233 flights) could be used to estimate a summer season arrival time density function for the Atlanta - Brussels flight. Data on 53 winter season Chicago - Montreal - Brussels flights could be used to estimate the proper density function.

2. Ramp maintenance manhour requirement density functions

In addition to the stochastic nature of aircraft arrival times, ramp maintenance manhour requirements are not deterministically known. In order to estimate, f(mh), the density function for the manhour requirements for the ramp maintenance activities performed during an aircraft's turnaround time, 69 data values could be retained as valid (for details, see [1]). It was our intent to examine four factors which are expected to have a direct effect on the ramp maintenance manhour requirements : the weather conditions, the maintenance crew foreman, the period of the day (morning or evening) and the aircraft turnaround time.

Ramp maintenance activities are performed at the boarding gate where part of the job requires activities to be performed outside the aircraft. As no valid data were available, the effect of the weather conditions on the working pace could not be investigated. Experience seems to suggest that the maintenance crew foreman is an important factor with respect to working pace. Unfortunately again, a lack of data forced us to neglect this factor. There was no actual need to trace the effect of morning or evening work, because all B747 aircraft transit through the home base during morning hours. It is interesting to note, however, that a test performed on available data for DC10 flights indicated no difference in the coefficients of two linear regressions - one for daytime and one for nighttime - of expected manhour requirements as a function of grounding time using the Chow test [2] with $\alpha = 25$ %. As for the aircraft turnaround time, a linear regression indicated the slope not to be significantly different from zero, not even at the 50 % level. Our analysis leads to the conclusion that a single manhour requirement density function, f(mh), may be estimated.

3. Estimating ramp maintenance manpower requirement density functions

If the assumption could be made that both aircraft arrival times and departure times and the manhour requirements are known deterministically, the exact ramp maintenance manpower could be determined using an integer linear programming model similar to the one developed in [6]. The fact that both arrival times and manhour requirements are stochastic in nature calls for a different type of analysis. We determine the ramp maintenance manpower requirements density function, f(m), on the basis of a careful analysis of the arrival time data and manhour requirement data for the two possible situations where one or two aircraft may be handled during a shift.

The analysis for the one aircraft per shift case poses no major problems. Actual experience indicates that a ramp maintenance work package consists of a series of unordered independent tasks. In addition it was already mentioned above that manhour requirements and arrival times are not correlated. Given the arrival time density functions, f(ATA), and the ramp maintenance manhour requirement density function, f(mh), derived according to the above mentioned procedures, let us assume there are $\underline{a}$ arrival time data values and $\underline{h}$ ramp maintenance manhour requirement data values available for the particular aircraft turnaround under consideration. We can now compute a total of $\underline{a.h}$ data values for the number of ramp maintenance workers required, according to the equation

$$m_{ij} = \left\lceil \frac{mh^j}{STD - ATA_i} \right\rceil , \qquad \begin{array}{l} 1 \leq i \leq a \\ 1 \leq j \leq h \end{array} \qquad (1)$$

where

mh^j = the jth manhour requirement data value, $1 \leq j \leq h$, available from f(mh) for the turnaround under consideration.

STD = the deterministically scheduled aircraft departure time, marking the end of the turnaround interval.

ATA_i = the ith actual arrival time data value, $1 \leq i \leq a$, available from f(ATA) for the turnaround under consideration.

$\lceil x \rceil$ = the smallest integer value greater than or equal to x.

The analysis for the situation where the two jumbo aircraft are handled per shift is complicated by the fact that the corresponding aircraft turnaround times may show a partial or complete overlap. Given the fact that arrival times and manhour requirements are uncorrelated and based on the assumption that maintenance manpower can be easily transferred between the two aircraft, we show in [1] that a total of (ah)x(a'h') data values for the number of ramp mantenance workers required can be determined as

$$m_{ii'jj'} = \max\left\{\left\lceil \frac{mh_1^j}{T_1^i} \right\rceil , \left\lceil \frac{mh_2^{j'}}{T_2^{i'}} \right\rceil , \left\lceil \frac{mh_1^j + mh_2^{j'}}{T_1^i + T_2^{i'} - T_0^{\ell}} \right\rceil\right\}, \quad \begin{array}{l} 1 \le i \le a \\ 1 \le i' \le a' \\ 1 \le j \le h \\ 1 \le j' \le h' \end{array} \qquad (2)$$

where

mh_1^j = the jth manhour requirement data value for aircraft 1, $1 \le j \le h$, for the considered turnaround.

$mh_2^{j'}$ = the j'th manhour requirement data value for aircraft 2, $1 \le j' \le h'$, for the considered turnaround.

T_1^i = the ith turnaround time data value for aircraft 1 defined as $T_1^i = STD_1 - ATA_1^i$, where STD_1 is the scheduled departure time for aircraft 1 and ATA_1^i denotes the ith arrival time data value for aircraft 1, $1 \le i \le a$, for the turnaround under consideration.

$T_2^{i'}$ = the i'th turnaround time data value for aircraft 2 defined as $T_2^{i'} = STD_2 - ATA_2^{i'}$, where STD_2 is the scheduled departure time for aircraft 2 and $ATA_2^{i'}$ denotes the i'th arrival time data value for aircraft 2, $1 \le i' \le a'$, for the given turnaround.

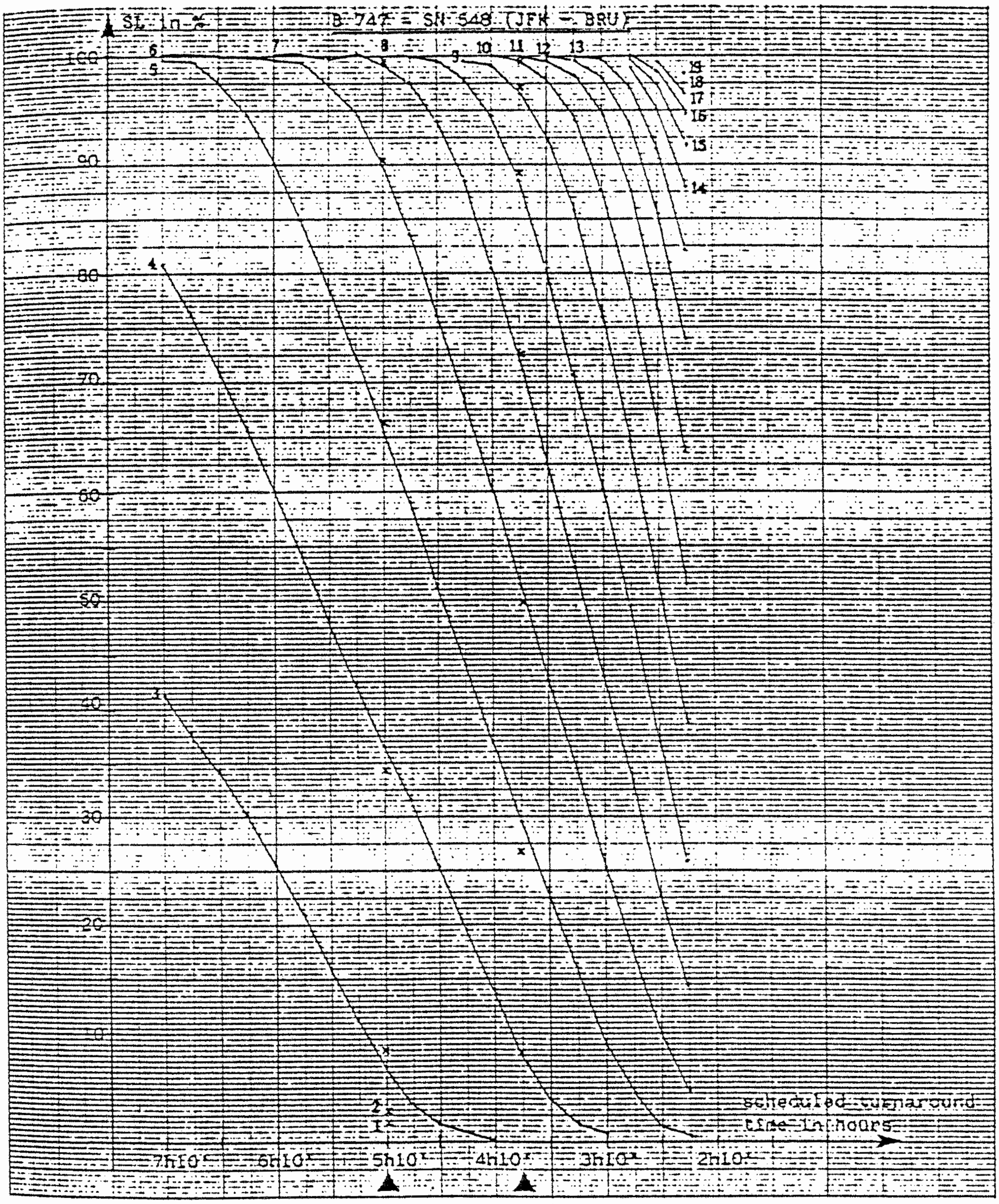
T_0^{ℓ} = the ℓ-th data value for the period during which the turnaround times of the two aircraft show an overlap with $1 \le \ell \le a.a'$.

$\lceil x \rceil$ = the smallest integer value greater than or equal to x.

4. Determining the ramp maintenance manpower requirements for a given service level

The procedure described above may be used to derive the ramp maintenance manpower requirement density functions, f(m), which are applicable to a certain flight. The next step is to compute the ramp maintenance manpower requirements for a given service level, SL, which is defined as the probability that a particular flight will depart on schedule. With that purpose curves can be derived, as indicated in figure 2, which for each flight give the manpower requirements for various values of the service level and the scheduled turnaround time. Figure 2 gives the corresponding results for the New York - Brussels flight on Tuesday and Friday mornings for the winter season 1984/85. As can be seen, a total of 7 ramp maintenance workers will guarantee a service level of 99.5 % on Tuesday morning, where 9 workers will guarantee a service level of 96.5 % on Friday morning. Figure 2 also shows the sensitivity of the results to changes in the number of manhour data values used. Using 69 data values for the manhour requirement instead of 44 (a 57 % increase) changes the guaranteed service level by only .5 % for SL > 50 %. Table I gives the ramp maintenance manpower requirements for the flight schedule of Figure 1, i.e. the number of maintenance workers required each day of the week and for various values of the service level.

It should be understood that considerable flexibility improvements in the day-to-day maintenance management may be obtained if reliable advance information can be gathered concerning the aircraft arrival times. Very often the maintenance supervisor can be informed by telex, before the start of the shift, about the expected arrival time of an aircraft. To illustrate the beneficial effects generated by this advance telex information, we need Figure 3 which plots the manpower requirements for various service level values and known turnaround times and which was obtained using the procedure described above but taking actual arrival times as input instead of the proper f(ATA) values. Consider now the New York - Brussels flight on Friday morning. According to Table I and Figure 2, 9 ramp maintenance workers are required to assure a departure on schedule



Wi 84/85, Tu.
ETD = 12h00'
ETA = 6h50'
T = 5h10'

Wi 84/85, Fr.
ETD = 10h45'
ETA = 6h50'
T = 3h55'

.:44 data points for mh
46 data points for arrivals
×:69 data points for mh
46 data points for arrivals

Figure 2 : SL = f(STT/m) for N.Y. return flight on B747.

TABLE I : Daily ramp maintenance manpower requirements for the flight schedule of Figure 1
(Wi 84/85 - 44 data points mh)

Day \ SL	95 %	90 %	85 %	80 %	75 %	70 %	65 %	60 %	55 %	50 %	45 %	40 %
Mo	13	12	11	10	10	10	9	9	9	8	8	8
Tu	7	6	6	6	6	6	5	5	5	5	5	5
We	11	10	10	10	10	9	9	9	9	9	8	8
Th	13	12	11	10	10	10	9	9	9	8	8	8
Fr	9	9	8	8	8	7	7	7	7	6	6	6
Sa	13	12	12	12	11	11	11	10	10	10	10	9
Su	11	10	10	10	10	9	9	9	9	9	8	8

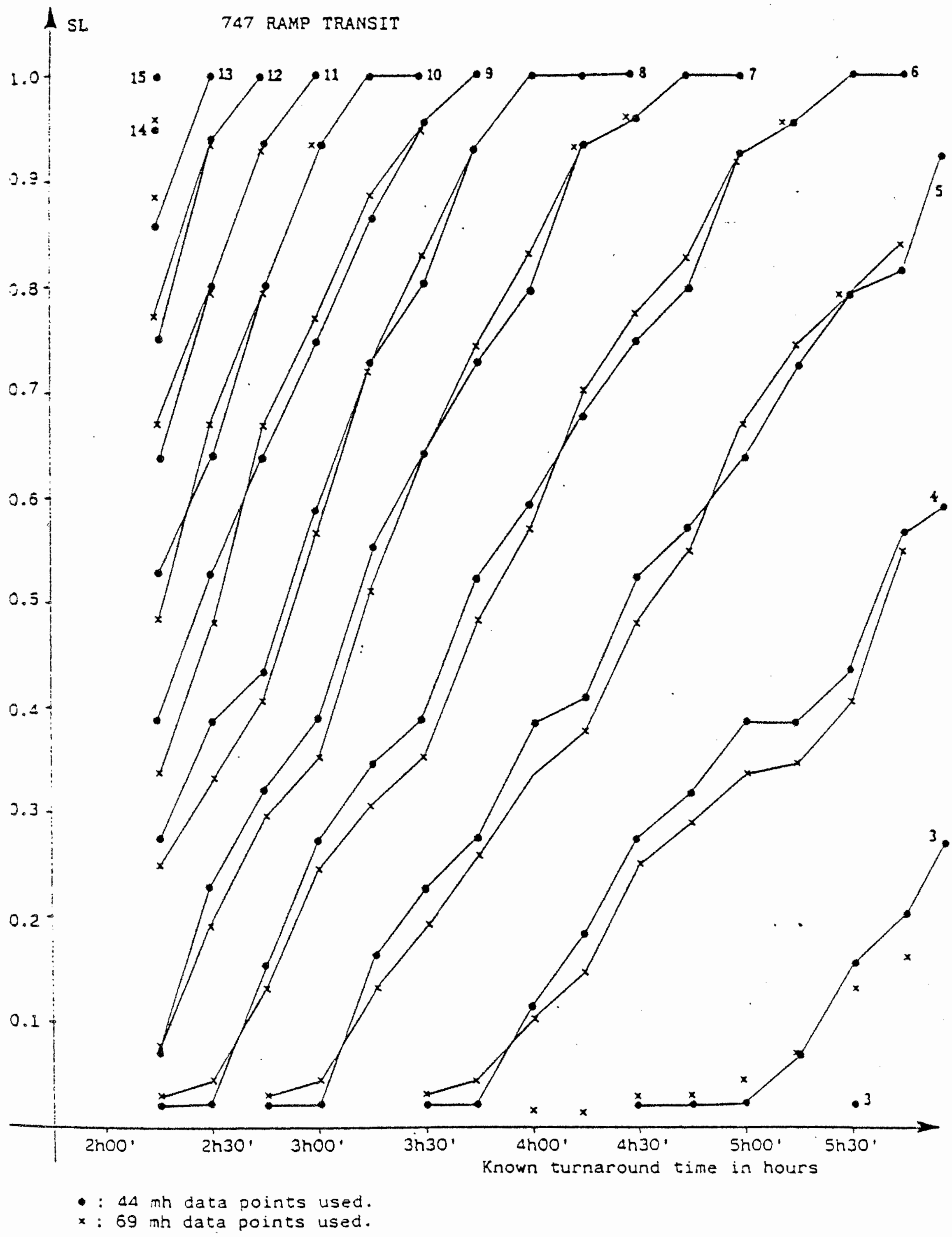


Figure 3 : SL = f(ATT/m) for B747

in 95 % of the cases. Assume now that the maintenance supervisor is informed by telex that on that particular Friday morning the flight is exactly on schedule such that the actual turnaround time is 3 h 55'. Figure 3 indicates that the job can be handled by 8 workers, allowing one worker to be shifted to another job. Table II gives the manhour savings which result when aircraft arrivals occur before the scheduled arrival time and when the workers thus saved can be transferred to other jobs. It can be noticed from the table that the manpower savings increase

TABLE II : Maximum manpower savings resulting from early arrivals

FLIGHT			$m^m_{.95}$	Δm	Gain mh (BF)	$m^m_{.6}$	Δm	Gain mh (BF)
N°	From	On						
570	Montreal	Monday Thursday	13	4	14.33 (11,908)	9	2	7.17 (5,958)
548	New York	Tuesday	7	1	5.90 (4,903)	5	1	5.90 (4,903)
		Friday	9	2	9.30 (7,728)	7	-	-

when service level goes up. Table III gives an other illustration of the interesting decision support information which results from the analysis performed so far. It indicates the maximum aircraft arrival delay while still guaranteeing a given service level. As can be seen, this maximum allowable delay decreases when service level goes down. Again this is valuable information for the maintenance supervisor in his efforts to meet scheduled departure times.

In deriving Eq. (2) it was argued that aircraft arrival times are independently distributed random variables (it is not because the New York return flight is late that the Atlanta return flight must be delayed also).

TABLE III : Maximum allowable arrival time delays for a given service level

FLIGHT			$m^m_{.95}$	Max. delay	$m^m_{.60}$	Max. delay
No	From	On				
SN570	Montreal	Monday Thursday	13	40'	9	25'
SN548	New York	Tuesday Friday	7 9	42.5' 27.5'	5 7	17.5' 30'

Furthermore, the maintenance manhour requirements for both aircraft are also independent (it is not because the aircraft coming in from New York suffers with a number of irregularities that the aircraft of the Atlanta return flight will also require a lot of trouble shooting and maintenance). Given the realistic assumption that maintenance manpower can be easily transferred between two aircraft, it is quite possible in practice that the same maintenance crew first finishes an early aircraft requiring few manhours and then readies the late aircraft. The benefits which may be obtained by this kind of manpower pooling are illustrated in Table IV for Saturday morning arrivals. The manpower requirements for flight SN548 from JFK are derived from Figure 2. The requirements for flight SN570 are taken from [1] and are derived in a similar way. The ramp maintenance manpower requirements indicated in the row labelel "combined" have been taken from Table I.

The analysis performed so far, also allows to obtain sufficient insight into the ramp maintenance spare manhours. Table V gives the spare manhours for each day of the week using a service level SL = 95 %. The rules behind the computation of the spare manhours can be illustrated by examining the figures in the first column for return flights on Monday morning. Assuming shift overlap time as non-productive one obtains a total

TABLE IV : Manpower requirement gain through pooling for Saturday morning flights

Flight		Ref.	Service levels												Scheduled turnaround time
No.	From		95	90	85	80	75	70	65	60	55	50	45	40	
SN 548	JFK	(from Fig.5)	9	8	8	8	7	7	7	7	6	6	6	6	4h05'
SN 570	YMX		13	12	11	10	10	10	9	9	9	8	8	8	3h05'
Sum			22	20	19	18	17	17	16	16	15	14	14	14	
Combined (data given in Table I)			13	12	12	12	11	11	11	10	10	10	10	9	5h10'
Gain			9	8	7	6	6	6	5	6	5	4	4	5	

of $7\frac{1}{3}$ available hours during the morning shift. Given a maintenance manpower requirement of 13 workers, one obtains a shift capacity of $(13)\ (7\frac{1}{3}) = 95.33$ manhours. From the manhour requirement density function, f(mh), one obtains an average manhour requirement of 22.59 manhours, which gives us 95.33 mh - 22.59 mh = 72.74 mh to spare during the morning shift, or 3782.48 mh on a yearly basis. This number is split up according to the manhours saved from the end of the night shift till the earliest aircraft arrival (35.75 mh), from the latest departure till the start of the afternoon shift (13 mh) and during the aircraft's turnaround time (23.99 mh).

5. Deriving the hangar maintenance manpower requirements

As mentioned above the majority of light maintenance checks must be performed at the maintenance hangar. It appears from the flight schedule of Figure 1 that hangar maintenance is possible from Monday morning till Tuesday noon and from Thursday morning till Friday noon.

It is the objective of the maintenance department to have the aircraft in the maintenance hangar one hour and a half beyond the scheduled arrival time. If the removal of the cargo, remaining catering and other cleaning activities would take too long, maintenance will have the aircraft towed away and will finish the work at the hangar. Two hours before the scheduled departure time all hangar maintenance work must be finished in order to have the aircraft at the ramp one hour and a half before the scheduled departure time. Taking into account breaks and shift overlaps, this leaves a fairly constant hangar maintenance job window length of 15 h 20' starting Monday morning and of 16 h 25' starting on Thursday morning. Due to hangar space constraints, either one of these two periods will have to be selected to perform the hangar maintenance activities.

Table VI gives an overview of the minimum manhour requirements for the light maintenance checks. Experience indicates that these requirements are fairly constant. The C and 2C checks mentioned in Table VI actually

TABLE V : Spare manhours required to guarantee a SL = 95 %

Period / Requirement	Mo	Tu	We	Th	Fr	Sa	Su	Weekly total
Before earliest arrival	35.75 (1859.00)	3.03 (157.56)	1.83 (95.16)	35.75 (1859.00)	3.90 (202.80)	5.63 (292.76)	1.83 (95.16)	87.72 (4561.44)
After latest departure	13.00 (676.00)	7.00 (364.00)	11.00 (572.00)	13.00 (676.00)	20.25 (1053.00)	13.00 (676.00)	11.00 (572.00)	88.25 (4589.00)
Long term average during grounding	23.99 (1247.48)	18.71 (972.92)	22.66 (1178.32)	23.99 (1247.48)	19.26 (1001.52)	31.52 (1639.04)	22.66 (1178.32)	162.79 (8465.08)
	72.74 (3782.48)	28.74 (1494.48)	35.49 (1845.48)	72.74 (3782.48)	43.41 (2257.32)	50.15 (2607.80)	35.49 (1845.48)	338.76 (17615.52)
Workers required or $m^{m}_{.95}$	13	7	11	13	9	13	11	

TABLE VI : Light maintenance requirements
Yearly flight hours : 5060

Check type	Interval	Efficiency in %	Yearly frequency	Spacing in days	Min. mh requirement	Yearly manhours
A	380 hours	87	16	23	72	1152
B	1400 hours / 6 months	93	4	93	378	1512
2B	2800 hours / 12 months	93	2	187	342	684
C	13 months	-	1	365	1554	1554
2C	26 months	-	1/2	365	1597	798
TOTAL YEARLY MANHOURS REQUIRED FOR REGULAR CHECKS :						5700
EXTRA WORK						9289
TOTAL NUMBER OF FLIGHT MAINTENANCE MANHOURS REQUIRED PER YEAR						14989

consist of four parts (P1, P2, P3, P4 and 2P1, 2P2, 2P3 and 2P4 respectively) with part P2 having the largest minimum manhour requirement of 460 manhours. As a result, 460 manhours must be available on Monday/Tuesday or Thursday/Friday. It is estimated that an additional 9289 manhours need to be available per year to cope with extra maintenance work (modifications, special inspections, component replacements, engine changes, etc.).

The next step is to set up a set of equations which will translate the manhour requirements into a number of workers that will be used during a particular shift on a particular day at a particular location. Let the decision variable R_{ijk} denote the number of maintenance workers needed on day i (i=1,2, ..., 7), during shift j (j=1,2 for the morning and afternoon shift) at location k (k=1,2 for the hangar and ramp). The 460 hangar manhour requirement for hangar visits starting on Monday or Thursday can now be expressed as

$$\sum_i \sum_j \sum_k a_{ijk} R_{ijk} \geq 460 \qquad (3)$$

where the coefficients a_{ij1} denote the computed number of hours to be spent at the hangar on day i, shift j. The detailed derivation of the a_{ij1} coefficients is fully explained in [1]. The total light maintenance manhour requirement of 14989 manhours (see Table VI) can be expressed in a similar way (for computational details, see [1]), leading to a single constraint of the type

$$\sum_i \sum_j \sum_k a_{ijk} R_{ijk} \geq 800 \qquad (4)$$

III. SOLVING THE MAINTENANCE CREW ALLOCATION PROBLEM

The purpose of this section is to set up an integer programming model which will allow to minimize overall maintenance manpower costs by deciding on the optimum number of maintenance crews to be used on each of the valid work and recreation schedules.

1. Deriving all valid work and recreation schedules

A work and recreation schedule for a particular worker will specify the shift (morning or afternoon) and off-duty assignments for each day of the week. Such a work and recreation schedule must satisfy a number of constraints due to legal obligations and union agreements. Figure 4 lists all work and recreation schedules which satisfy two constraints stemming from legal obligations and union requests specifying that a "working week" consists of five working days and that a worker should have two consecutive days off. Figure 5 lists the six schedules which do not satisfy the two days off requirement, but which are, however, still acceptable to the workers' unions. The 62 allowable work and recreation schedules can be identified by a matrix $\underline{B}$ consisting of 62 columns, corresponding to the 62 allowable schedules, and 14 columns, corresponding

m = morning shift
a = afternoon shift
o = off duty
X_j, j=1,...,57 denotes the decision variable for the number of workers assigned to work and recreation schedule j.

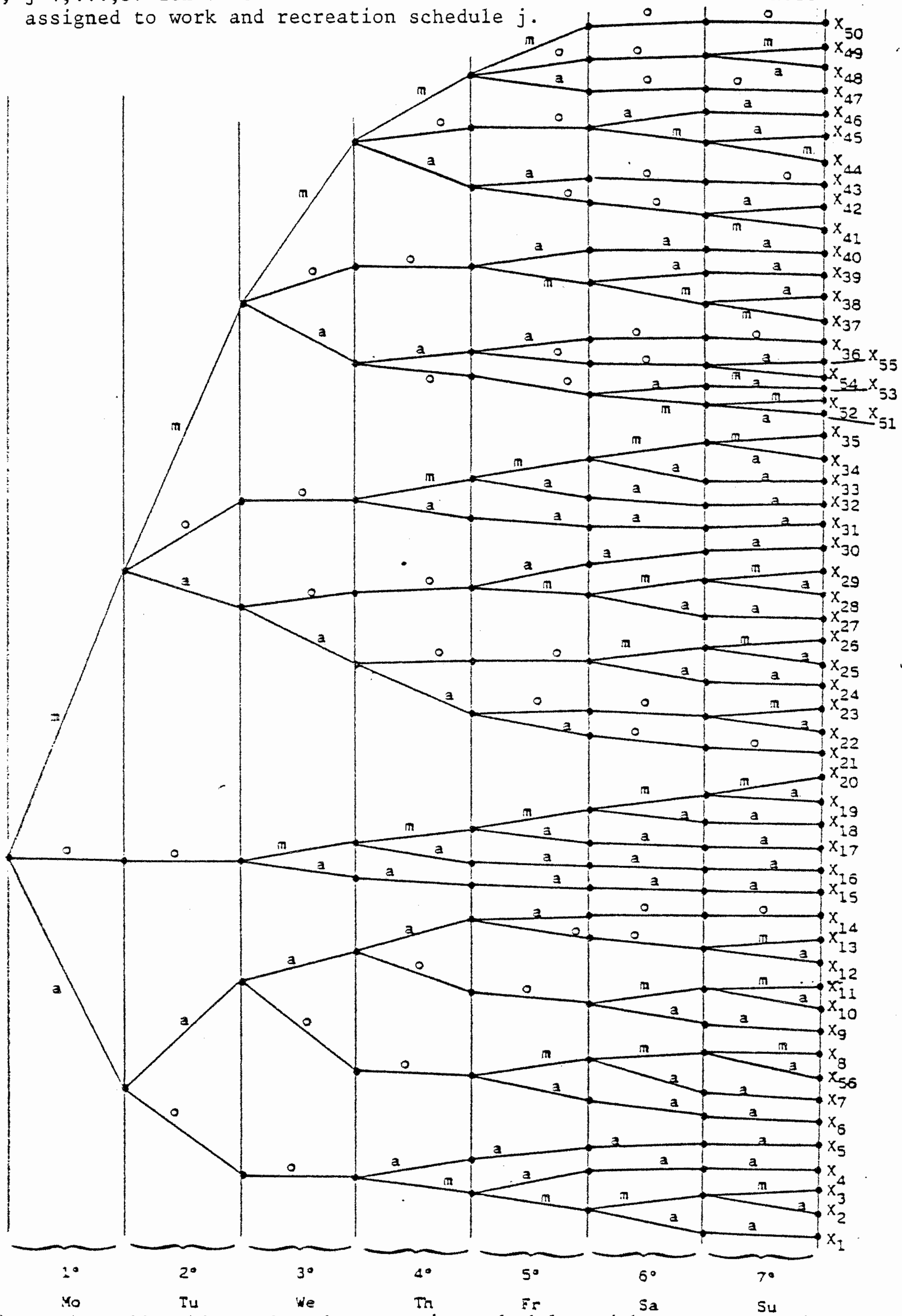


Figure 4 : Allowable work and recreation schedules with two consecutive days off

m = morning shift
a = afternoon shift
o = off duty
X_j, j=57...62 denotes the decision variable for the number of workers assigned to work and recreation schedule j.

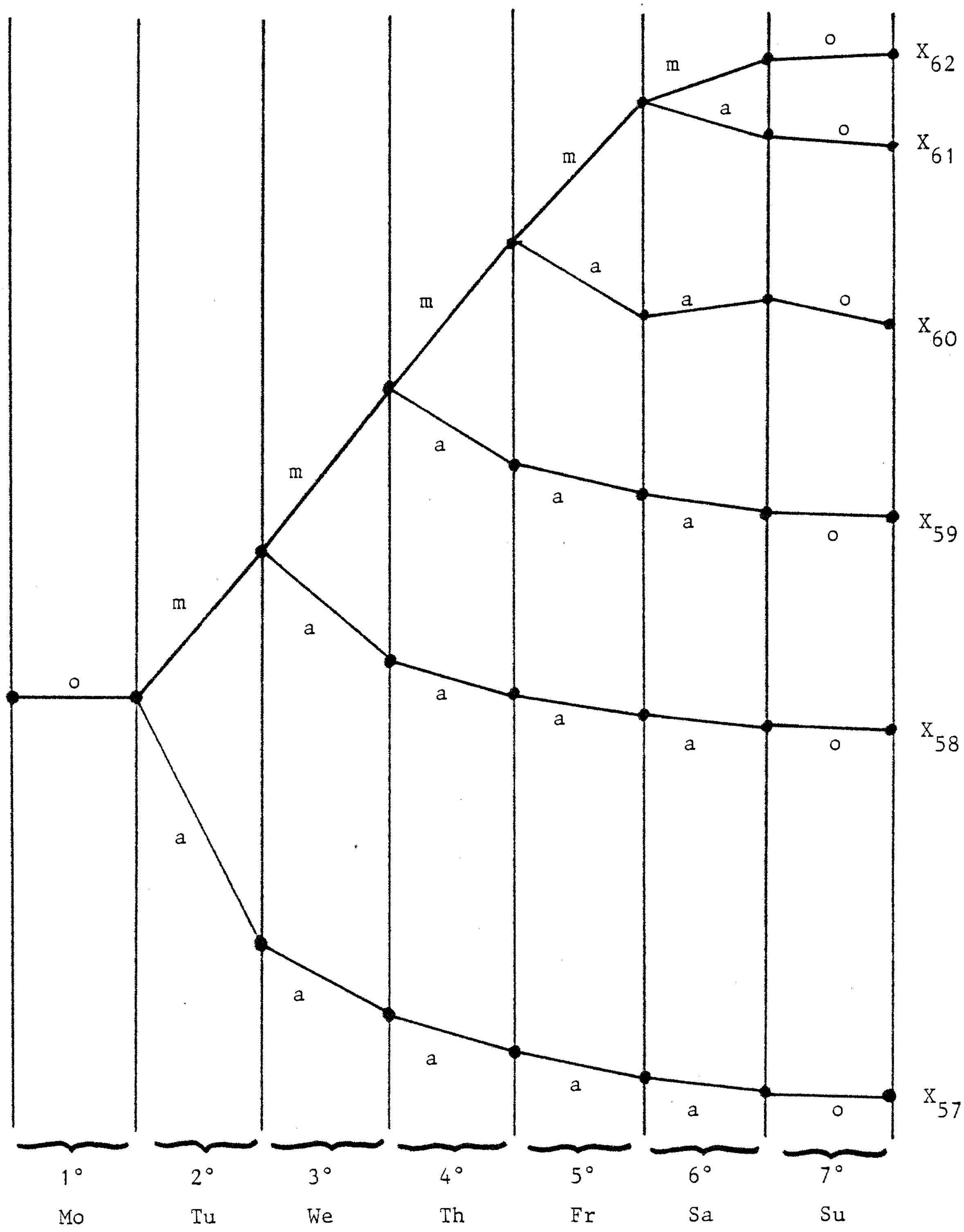


Figure 5 : Additional work and recreation schedules acceptable to labor unions for the department concerned.

to the 7 possible morning shifts and the 7 possible afternoon shifts during a week. An element b_{ij} will then equal 1 if a worker having recreation schedule j is on duty during shift i. Each column of matrix $\underline{B}$ will contain 5 ones corresponding to the 5 working shifts and 9 zeroes corresponding to the 9 shifts that a worker is off duty during a week.

2. The basic ILP formulation

Let

$\underline{C}$ = a row vector where element C(j) indicates the weekly cost per worker having work and recreation schedule of type j (j=1,2,...J where J = 62 in our example).

$\underline{B}$ = the matrix of allowable work and recreation schedules as defined above.

$\underline{X}$ = a column vector where element X(j) indicates the number of workers assigned to work and recreation schedule of type j, j=1,2,...,J.

$\underline{R}_R$ = a column vector where element $R_R(i)$ indicates the number of workers to be assigned on shift i (i=1,2, ...I where I = 14 in our example) at the ramp.

$\underline{A}^{SL}$ = a column vector where element A(i) indicates the number of workers required at the ramp on shift i for the chosen service level, SL. The vector entries are computed according to the procedure explained in Section III.4.

$\underline{R}_H$ = a column vector where element $R_H(i)$ indicates the number of workers to be assigned on shift i at the maintenance hangar.

$$Z_1 = \begin{cases} 1 \text{ if hangar maintenance is assigned to the Monday/Tuesday job window} \\ 0 \text{ otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1 \text{ if hangar maintenance is assigned to the Thursday/Friday job window} \\ 0 \text{ otherwise} \end{cases}$$

$\underline{L}_R$ and $\underline{L}_H$ = vectors where elements $L_R(i)$, resp. $L_H(i)$, indicate the number of hours to be spent at the ramp, resp. the hangar during shift i for the Monday/Tuesday job window (see the a_{ijk} coefficients in Eq. (3)).

$\underline{T}_R$ and $\underline{T}_H$ = vectors where elements $T_R(i)$, resp. $T_H(i)$, indicate the number of hours to be spent at the ramp, resp. the maintenance hangar during shift i for the Thursday/Friday job window (see the a_{ijk} coefficients in Eq. (3)).

$\underline{O}_R$ and $\underline{O}_H$ = vectors, the elements of which correspond to the a_{ijk} coefficients used in Eq. (4).

The ILP model for determining the number of maintenance workers to be assigned to the allowable work and recreation schedules in order to minimize the weekly maintenance costs can now be written as

$$\text{Min} \quad C \quad X \tag{5}$$

subject to

$$BX \geq R_R + R_H \tag{6}$$

$$R_R \geq A^{SL} \tag{7}$$

$$L_R R_R + L_H R_H \geq 460 \; Z_1 \tag{8}$$

$$T_R R_R + T_H R_H \geq 460 \; Z_2 \tag{9}$$

$$Z_1 + Z_2 = 1 \tag{10}$$

$$O_R R_R + O_H R_H \geq 800 \tag{11}$$

$$X \geq 0 \text{ and integer} \tag{12}$$

$$Z_1, Z_2 = 0 \text{ or } 1 \tag{13}$$

The objective function Eq. (5) will minimize the weekly maintenance costs resulting from the ramp and hangar maintenance worker assignments to the selected allowable work and recreation schedules. The constraints

Eqs. (6) - (7) assure that the selected work and recreation schedule assignments satisfy the minimum maintenance manpower allocations at the ramp and the hangar for each shift. Eqs. (8-10) are introduced to meet the 460 manhour requirements at the hangar during the Monday/Tuesday or the Thursday/Friday job window (i.e. Eq. (3) in the previous section). Eq. (11) represents the total light maintenance manhour requirement (i.e. Eq. (4) in the previous section). As such, the above ILP model is in a sense similar to the basic model 2.2 used by Tien & Kamiyama [7] to categorize possible stage 2-problem formulations used in their five-stage manpower scheduling framework.

3. The 12-hour off-duty constraint between work and recreation schedules

The ILP-model given above guarantees that only allowable work/recreation schedules (i.e. basically satisfying the two consecutive days off constraint) are accepted in the solution. The work and recreation schedules, however, must still be combined into a suitable shift schedule (i.e. stage 5 in the five-stage framework of Tien & Kamiyama [7]). The shift schedule must also satisfy the regulations of the airline's Standing Order nr. 1006 (an outgrowth of Koninklijk Besluit No 225, Dec. 7, 1983) specifying that there should be at least a 12-hour off-duty period between any two working periods. This 12-hour off-duty period is respected within the work and recreation schedules (figure 4 and 5) for each period of 7 consecutive days. The same requirement, however, must also be satisfied between successive work and recreation schedules, i.e. from Sunday to Monday. As will be discussed in Section IV, this requirement will simplify the procedure for creating a rotating shift schedule. It will guarantee the creation of a feasible cyclical rotating schedule of weekly work and recreation schedules with a schedule period of length N, the total number of workers or crews, times I days; that is, every worker or crew of workers rotates through every one of the N subschedules, each of length I (that is 7 days in our case), and repeats this common schedule over time every N x I days.

As an example, consider work/recreation schedule number 48 of Figure 4 which starts with the morning shift on Monday and ends with the afternoon shift on Sunday. It is clear that this work and recreation schedule can never precede or succeed itself without violating the 12-hour off constraint. Let $\{(i,j)\}$ represent the set of all work and recreation schedules starting with shift i on Monday and ending with shift j on Sunday, where $i, j \varepsilon \{m,a,o\}$ with m = morning shift, a = afternoon shift and o = off duty. It is shown in [1] that the subset of work and recreation schedules $N_a = \{(ao), (oa), (am), (om)\}$ will always satisfy the 12-hour off-duty constraint when combined into a shift schedule. This is easily illustrated by the network of Figure 6. The nodes denote the four work and recreation schedules involved, the directed arcs (ij) indicate that schedule i may precede schedule j without violating the 12-hour off constraint. As can be seen, a cycle exists between any possible combination of nodes indicating that the 12-hour constraint can always be met.

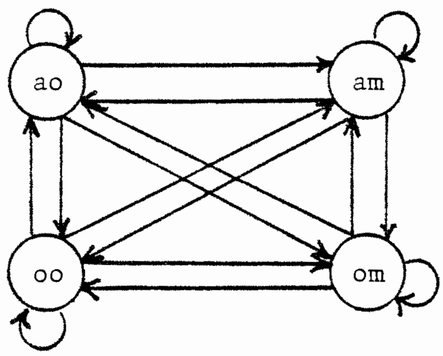


Figure 6 : The set N_a of work and recreation schedules

The remaining subset of work and recreation schedules $N_b = \{(mo), (ma), (mm), (aa), (oa)\}$ can only be sequenced under certain conditions. It is shown in [1] that if no worker has a work and recreation schedule belonging to N_a, the 12-hour off-duty constraint requires that no worker be given a schedule of type (ma), and that schedules of type (mo) combined with type (mm) cannot be used in combination with schedules of type (aa) combined with type (oa). If at least one worker has a schedule of the type belonging to set N_a, then it is shown in [1] that the 12-hour off duty constraint requires that the number of workers having a schedule of type (ma) may not exceed the number of workers allocated to schedules of the type specified by the set N_a.

The 12-hour off duty requirement can now be introduced by adding the following additional set of constraints to the basic ILP model of Eqs. (5)-(13) above :

$$\sum_{j \in N_a} X_j \geq Z_5 \tag{14}$$

$$\sum_{j \in N_c} X_j \leq MZ_3 + MZ_5 \tag{15}$$

$$\sum_{j \in N_d} X_j \leq MZ_4 + MZ_5 \tag{16}$$

$$Z_3 + Z_4 = 1 \tag{17}$$

$$\sum_{j \in N_e} X_j \leq \sum_{j \in N_a} X_j \tag{18}$$

where

$$Z_k = 0 \text{ or } 1 \quad \text{for } k = 3,4,5 \tag{19}$$

In this formulation, M is a sufficiently large number, the decision variables X_j denote the number of workers assigned to work and recreation schedule of type j and $N_a = \{(ao), (oa), (am), (om)\}$, $N_c = \{(mo), (mm)\}$, $N_d = \{(aa), (oa)\}$ and $N_e = \{(ma)\}$.

4. The crew problem

The ILP formulation given above yields the number of maintenance workers to be allocated to each work and recreation schedule. The total number of workers per week is then $\sum_j X_j = N$. As already suggested above the individual schedules can now be combined into a rotating schedule where each worker repeats this schedule every N weeks. As an example assume the ILP-solution yields $X_{37} = 2$ and $X_{50} = 3$ such that $N = 5$. A possible rotating schedule is illustrated in the table below, where the entries refer to the work and recreation schedule numbers :

worker \ week	1	2	3	4	5
1	37	50	50	50	37
2	37	37	50	50	50
3	50	37	37	50	50
4	50	50	37	37	50
5	50	50	50	37	37

As can be seen workers 1, 2 and 3, 4, 5 are on the same schedule the first week; workers 2, 3 and 1, 4, 5 are on the same schedule the second week, and so on. Workers and their unions strongly resist this type of cyclical schedules where a worker's working mates may change every week. The stylish way to obviate this problem is the formation of crews working together all the time. Selecting the optimal size of those crews is an important, but cumbersome problem. The way we go around with it, is to rewrite the ILP model in crew terms, allowing it to be solved for various realistic crew size values.

Let

$\underline{X}^c$ = a column vector where element X_j^c now indicates the number of crews assigned to work and recreation schedule of type j.

$\underline{C}^c$ = a row vector where element C_j^c indicates the weekly cost per crew having work and recreation schedule of type j ; i.e., $C^c = (CS)C$ where CS is the selected crew size and vector $\underline{C}$ as defined above.

$\underline{B}^c$ = (CS)B, with $\underline{B}$ as defined earlier.

Keeping the other vector definitions unchanged, the ILP model may now be rewritten as :

$$\text{Min} \quad C^c \; X^c \tag{20}$$

subject to

$$B^c X^c \geq R_R + R_H \tag{21}$$

$$R_R \geq A^{SL} \tag{22}$$

$$L_R R_R + L_H R_H \geq 460 \; Z_1 \tag{23}$$

$$T_R R_R + T_H R_H \geq 460 \; Z_2 \tag{24}$$

$$O_R R_R + O_H R_H \geq 800 \tag{25}$$

$$Z_1 + Z_2 = 1 \tag{26}$$

$$\sum_{j \varepsilon N_a} X_j^c \geq Z_5 \tag{27}$$

$$\sum_{j \varepsilon N_c} X_j^c \leq MZ_3 + MZ_5 \tag{28}$$

$$\sum_{j \varepsilon N_d} X_j^c \leq MZ_4 + MZ_5 \tag{29}$$

$$Z_3 + Z_4 = 1 \tag{30}$$

$$\sum_{j \varepsilon N_e} X_j^c \leq \sum_{j \varepsilon N_a} X_j^c \tag{31}$$

$$Z_k = 0 \text{ or } 1 \quad \text{for } k = 3,4,5 \tag{32}$$

$$X_j^c \geq 0 \text{ and, integer} \qquad j = 1,2, \ldots, J \tag{33}$$

As can be seen Eq. (21), in combination with Eq. (22), denote that the number of workers made available must be sufficient to meet ramp and hangar maintenance requirements. This assumes that crews may be split between ramp and hangar. If crew splitting is not allowed, it is sufficient to replace Eqs. (21) - (25) by the following :

$$B\,X^c \geq R_R^c + R_H^c \tag{34}$$

$$R_R^c \geq A_c^{SL} \tag{35}$$

$$L_R^c R_R^c + L_H^c R_H^c \geq 460\ Z_1 \tag{36}$$

$$T_R^c R_R^c + R_H^c R_H^c \geq 460\ Z_2 \tag{37}$$

$$O_R^c R_R^c + O_H^c R_H^c \geq 800 \tag{38}$$

where

$$L_R^c = (CS)\ L_R \text{ and } L_H^c = (CS)\ L_H$$

$$T_R^c = (CS)\ T_R \text{ and } T_H^c = (CS)\ T_H$$

$$O_R^c = (CS)\ O_R \text{ and } O_H^c = (CS)\ O_H$$

and

R_R^c = a column vector where element $R_R^c(i)$ denotes the number of crews to be assigned on shift i at the ramp.

A_L^{SL} = a column vector where element $A_c^{SL}(i) = \left\lceil \frac{A^{SL}(i)}{CS} \right\rceil$

R_H^c = a column vector where element $R_H^c(i)$ denotes the number of crews to be assigned at the hangar during shift i.

5. Computational results

Experimentation with the two ILP-models using the integer programming capabilities of the LINDO software package, run on a IBM 3033 computer, could not provide an optimal solution within a one hour interactive session at the terminal. Our efforts to investigate and exploit possible structural characteristics of the ILP-models remained unsuccessful so far. Research is continued in this direction. In the meanwhile, the possibly dangerous decision was made to solve the models using an LP-procedure and to exploit the potential benefits of its sensitivity analysis.

In order to support a decision in favor of the Monday/Tuesday or the Thursday/Friday hangar maintenance job window, two LP models were solved for crew sizes varying from 1 to 7 workers; the first one of the type Min C^cX^c subject to $B\ X^c \geq R^c_R + R^c_H$, $R^c_R \geq A^{SL}_c$, $O^c_R R^c_R + O^c_H R^c_H \geq 800$ in combination with $L^c_R R^c_R + L^c_H R^c_H) \geq 460$ and the second one of the same type but in combination with $T^c_R R^c_R + T^c_H R^c_H \geq 460$.

Figure 7 illustrates the results. It is seen that a grounding on Thursday/Friday with a maximum CS = 7 is cheaper than a grounding on Monday/Tuesday with CS = 1 for all SL. Consequently only groundings on Thursday/Friday are considered in the remaining analysis. Figure 8 depicts the weekly cost of all B747 maintenance with Thursday/Friday grounding and no crew splitting allowed as a function of the service level for various crew sizes.
Figure 8 clearly shows how the minimum weekly cost increases with SL. On the other hand, Figure 9 shows that the minimum cost does not monotonously increases with the crew size for a given service level; even more than one local maximum may exist. The possible cost decrement following a local maximum increases with the crew size.
Table VII lists the optimal work and recreation schedules. As can be seen only four different schedules are in the optimal solution of which maximum three are used simultaneously. This indicates the need for an efficient dominance procedure for eliminating poor schedule candidates from the model. In addition, the schedules obtained only require morning work so that the 12-hour off duty constraints (Eqs. (27)-(31)) are satisfied.

Only four constraints of the set $B\ X^c \geq R^c_R + R^c_H$ were binding for crew size values equal to four workers or more. The 460 mh constraint is binding too. The dual cost value is a constant 2217 BF/week for all crew size and service level values. Figure 10 gives the sensitivity to the right hand side of the 460 mh constraint. The figure indicates that considerable savings can be obtained by reducing the 460 mh requirement. The overall 800 mh requirement was never binding.

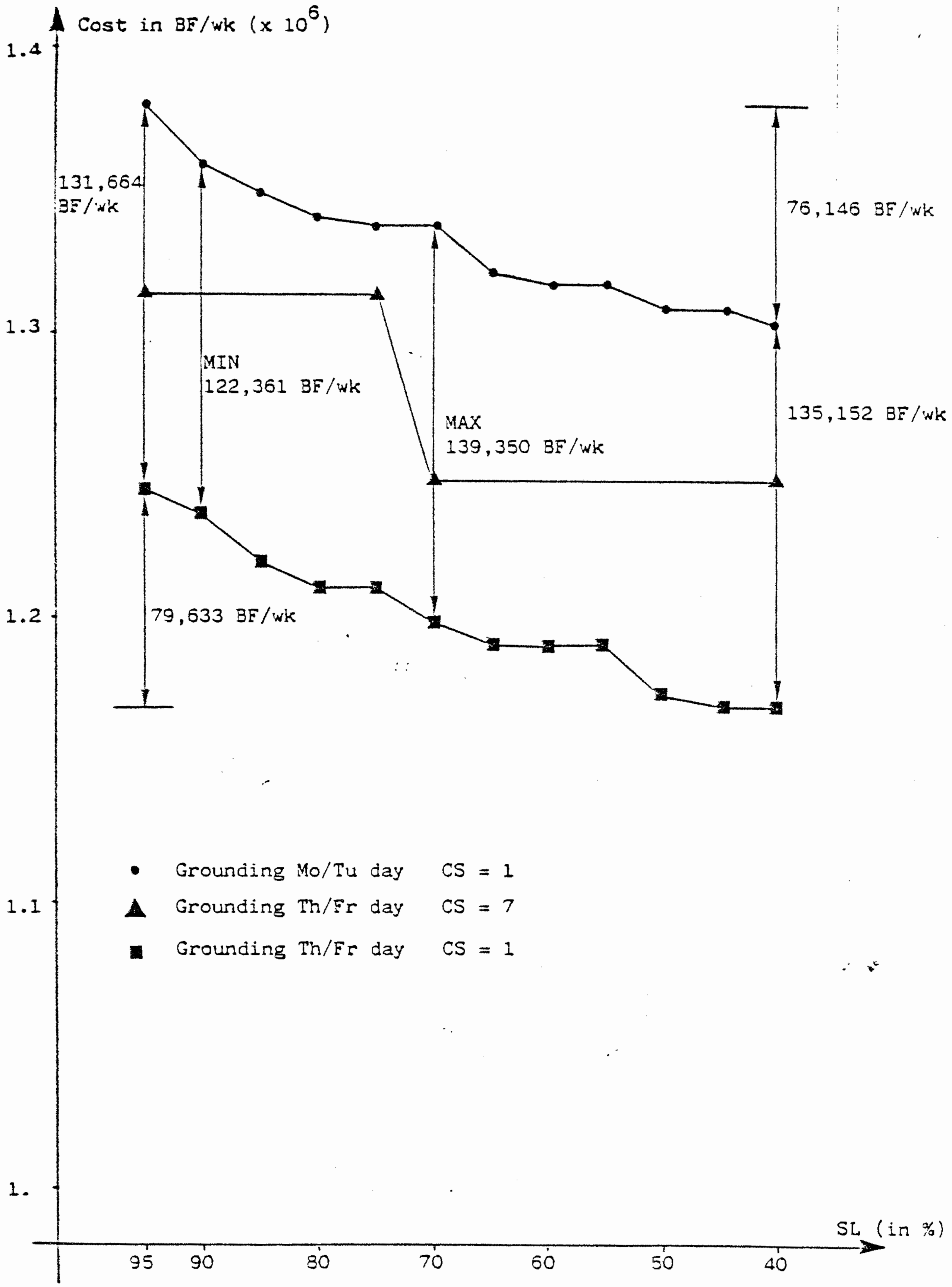


Figure 7 : Weekly cost of all B747 maintenance. Grounding on Monday/ Tuesday and on Thursday/Friday.

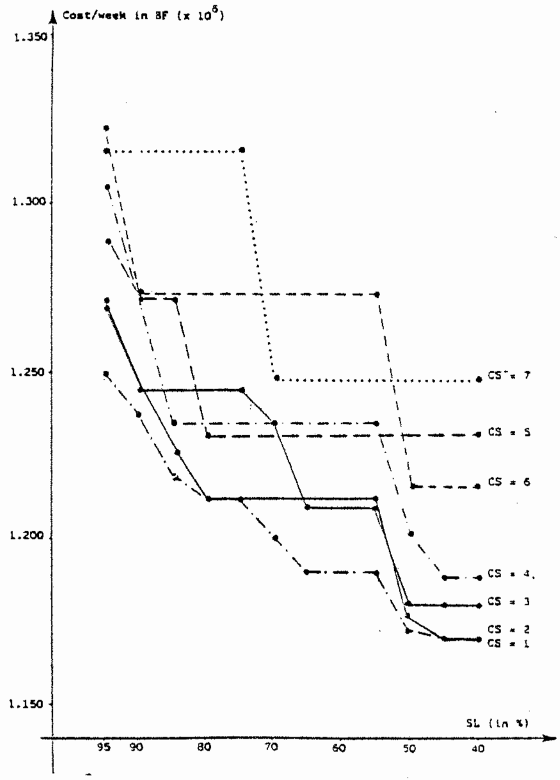


Figure 8 : Weekly cost of all B747 maintenance. Grounding on Thursday/ Friday and no crew splitting.

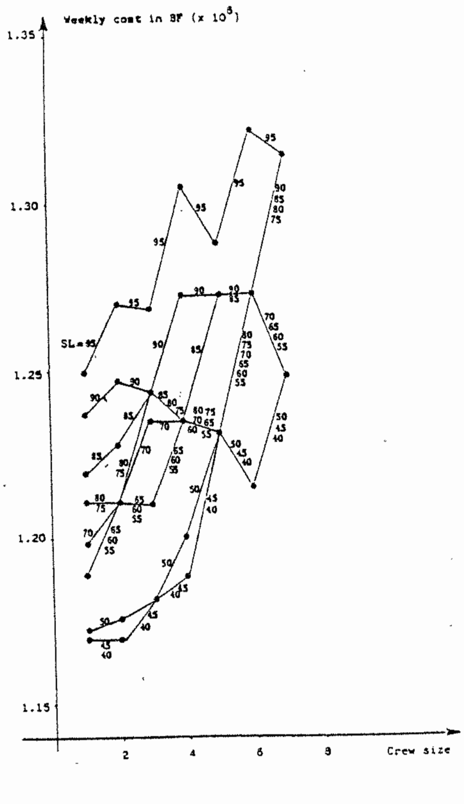


Figure 9 : Weekly cost of all B747 maintenance. Grounding on Thursday/ Friday and no crew splitting.

TABLE VII : All B 747 maintenance. Work and recreation schedules. Grounding on Thursday/Friday and no crew splitting.

Service	x_{20}						x_{35}						x_{50}							x_{62}					
	1	2	3	4	5	6	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6
95 %	11		4	3		2	6			3		2	13	1	5	4	9.41	3	7.08	36.74	23.81	11.52	8.85		5.7
90 %	10				2		5	4	3		2	2	12	1	16.11	12.44	3	8.3	7.08	38.33	24.41			7.41	
85 %	10				2		5	4	3		2	2	47.44	1	16.11	11.96	3	8.3	7.08	2	23.93			7.41	
80 %	10				2		5	4	3		2	2	47.03		16.11	11.96	9	8.3	7.08	2	24.52			1	
75 %	10				2		5	4	3		2	2	48.04		16.11	11.96	9	8.3	7.08	1	24.52			1	
70 %	9				2		5	3	3		2	2	17.64		16.11	11.96	9	8.3	6.60	31.92	24.52	1		1	
65 %	9				2		5	3	3		2	2	19.65		15.7	11.96	2	8.3	6.60	29.49	24.52	1		8	
60 %	9				2		5	3	3		2	2	20.71		15.7	11.96	10	8.3	6.60	28.43	24.52	1		0.90	
55 %	9				2		5	3	3		2	2	20.71		15.7	11.96	10	8.3	6.60	28.43	24.52	1		0.90	
50 %	9	1		3	2		4	3			2	2	47.26		15.22	11.56	10	7.81	6.60	1	23.63	1		0.90	
45 %	8			2	2		4	3			2	2	47.26		15.22	11.56	10	7.81	6.60	2	24.63	1	1	0.90	
40 %	8			2	2		4	3			2	2	48.26		16.22	11.56	10	7.81	6.60	1	24.63		1	0.90	

When crew splitting is allowed, i.e. the model given by Eqs. (20)-(25) but only solved for Thursday/Friday grounding, the results indicate that the minimum cost is no longer a function of the crew size. Indeed, for a given service level, the ramp maintenance requirements remain fixed over the entrie crew size range. Again, the same four work and recreation schedules (type 20, 35, 50 and 62) were only candidates for the optimal solution. They only require morning work which justifies the omittance of Eqs. (27)-(31) from the model since the 12-hour constraint will automatically be satisfied. Again considerable savings can be obtained by reducing the 460 mh requirement and the overall 800 mh constraint was never binding. Using parametric RHS programming, the 460 mh requirement was dropped in steps of 50 mh. The results indicate that a yearly cost saving of BF 29 051 568 can be obtained by reducing the 460 manhour requirement to 200 mh.

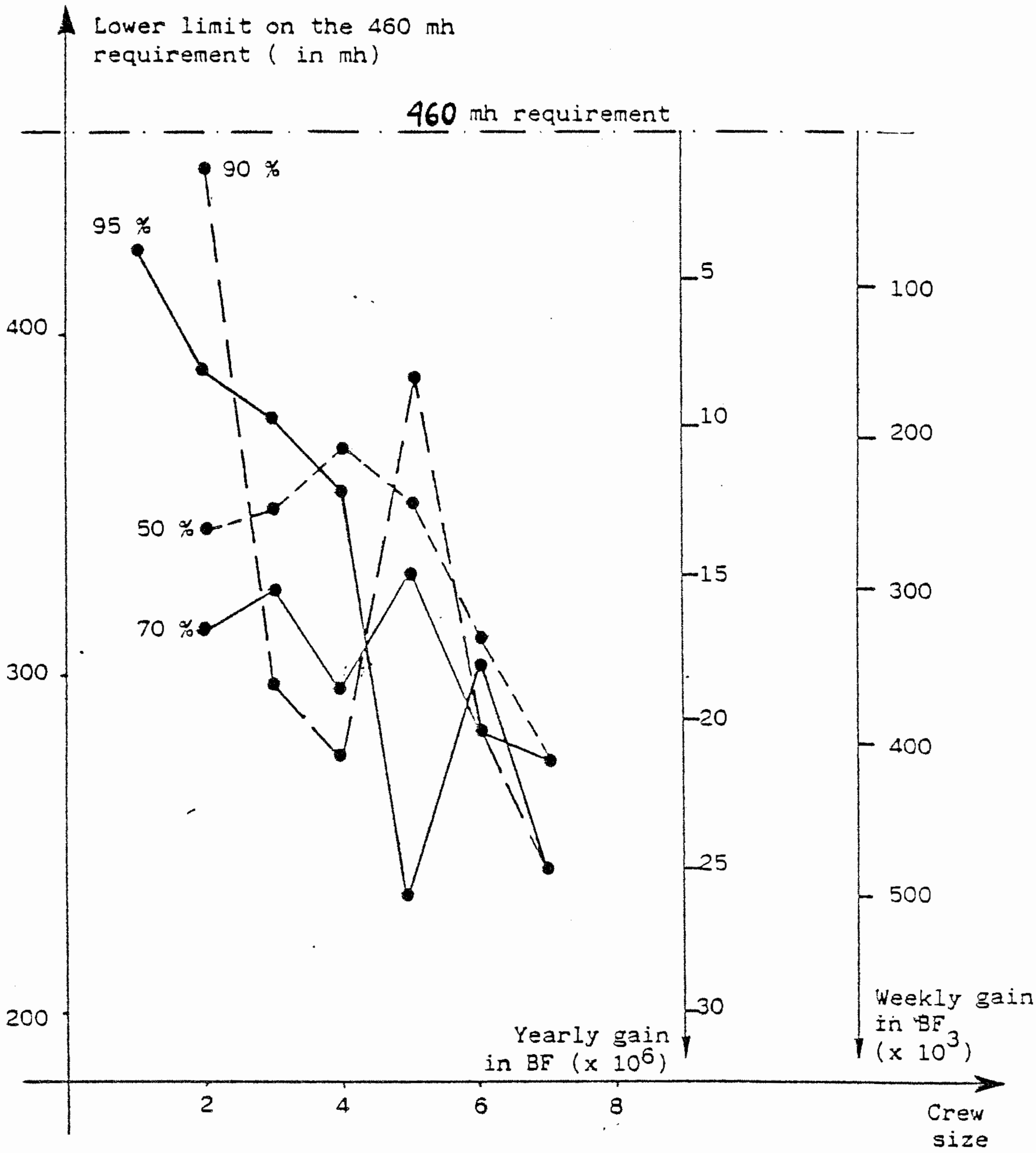


Figure 10 : Lower limit on the 460 mh requirement equation during grounding on Thursday/Friday.

IV. SEQUENCING THE WORK AND RECREATION SCHEDULES

The models discussed in the previous section, if optimally solved yield the number of maintenance crews to be allocated to allowable work and recreation schedules such that the weekly maintenance cost is minimized. Constraints have already been added to guarantee that the work and recreation schedules can be sequenced without violating the 12-hour off-duty constraint, always yielding a feasible rotating shift schedule where the crews repeat the schedule over time every N weeks (see Sections III.3 and III.4 above and also Stage 4 in the five stage framework discussed by Tien & Kamiyama [7]).

The resulting rotating schedules will be appreciated in a different way by the various maintenance crews. In order to illustrate this, assume again that $X_{37} = 2$ and $X_{50} = 3$ are in the optimal solution, yielding N = 5 workers. Two possible rotating schedules are given in the table below :

	week 1	week 2	week 3	week 4	week 5
Shift schedule example 1	type 50 mmmmmoo	type 37 mmoommm	type 37 mmoommm	type 50 mmmmmoo	type 50 mmmmmoo
Shift schedule example 2	type 37 mmoommm	type 50 mmmmmoo	type 37 mmoommm	type 50 mmmmmoo	type 50 mmmmmoo

As can be seen some crews will prefer shift schedule 1 which requires crews to work during 2 consecutive weekends (week 2-3 and week 3-4) and only has one 8 consecutive days working period, while others prefer shift schedule 2 which has two 8 consecutive days working periods but never requires crews to be on duty during consecutive weekends. The problem is to derive a common shift schedule which in a sense has the maximum "utility" to all maintenance crews.

The number of possible shift schedules to evaluate is given by $N!/\prod_{j\varepsilon S} X_j^c!$, where S is the set of work and recreation schedules in the solution, X_j^c the number of crews having work/recreation schedule of type j and $N = \sum_{j\varepsilon S} X_j^c$. Rather than estimating the utility of each shift schedule (which would require 5!(2!)(3!) worker evaluations in our example, we ask the workers to express their aversion towards each sequence of two work and recreation schedules in the solution, on an interval scale. An average aversion index value can thus be obtained for each sequence. A maximum of $|S|^2$ average aversion index values must thus be obtained if $X_j > 1 | j\varepsilon S$. For our example given above an aversion index must be obtained for the 4 sequences (types 37-37, type 50-50, type 37-50, type 50-37) which may occur in the possible shift schedules made up with work/recreation schedules of type 37 and 50. For each shift schedule the overall aversion index can then be computed as the sum of the aversion indices obtained for the schedule's individual work and recreation schedule sequences.

Obtaining the aversion index values is a difficult task (see also [8]). But if they can be obtained, they may be used to determine the rotating shift schedule which minimizes the workers' aversion. This problem can be modeled as the well-known asymmetrical traveling salesperson problem where each of the N work and recreation schedules which should appear in a rotating shift schedule is associated with a city and the average aversion index values correspond with the distances between the cities. Finding the best rotating schedule now reduces to the problem of finding the shortest tour.

As an example, Figure 11 gives the aversion matrix (i.e., the distance matrix for the traveling salesman problem) for the case of a crew size equal to 7 workers, a service level of 90 % and crew splitting allowed between ramp and hangar. The traveling salesman problem is NP-complete, justifying the use of heuristic solution methods. Figure 12 gives the results obtained with a variant of the well-known "nearest neighbour" method (fully described in [1]) which basically starts from a work and recreation schedule and successively adds the nearest neighbour schedule until a feasible rotating shift schedule is obtained. This procedure is repeated for each work and recreation schedule in the solution (so $|S|$ times) and the shift schedule with the lowest overall aversion is selected.

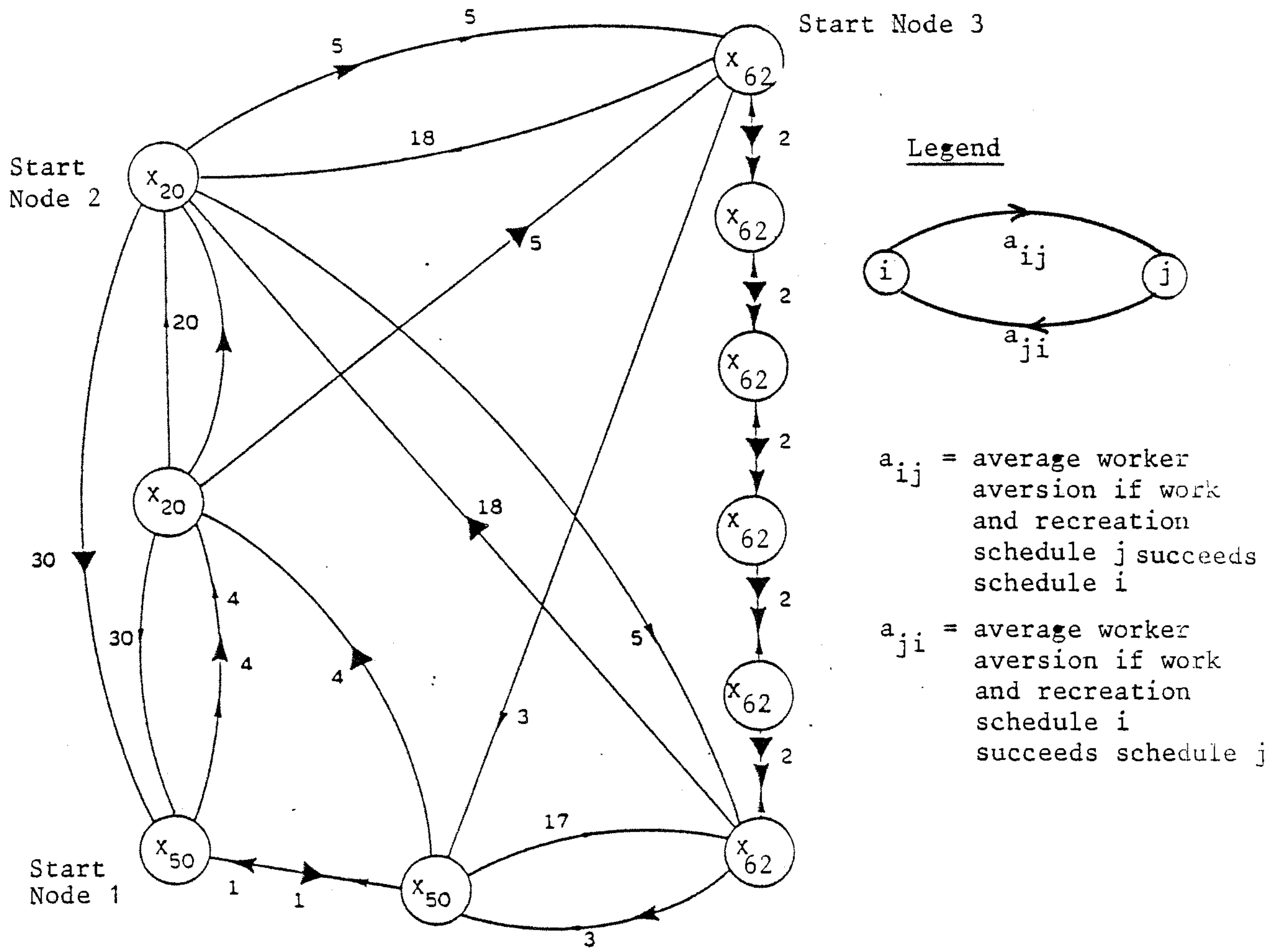
V. CONCLUSIONS

The present paper describes an integrated solution procedure for the scheduling of crews for line and light maintenance for a small fleet of aircraft operated on medium and long range continental routes. Coping with both the stochastic nature of aircraft arrival times and the manhour requirements for the various maintenance tasks, a procedure is described to derive overall manhour requirement and arrival time density functions which in combination with a proper service level choice allow to derive the minimum line and light maintenance manpower requirements for each shift. This procedure, a major departure from previous methods which usually ignore the statistical nature of aircraft arrivals and manhour requirements, requires a serious computational effort even for the two-aircraft example considered in this paper.

Spanning various stages of the five-stage manpower scheduling framework of Tien & Kamiyama [7], we set up an integer linear programming model to decide on the optimal number of maintenance crews to be used on each of the allowable work and recreation schedules satisfying the various maintenance manpower requirements and specific constraints resulting from labor contract specifications. Due to the computational effort involved, computational results were only obtained with all integer and 0/1 constraints removed. Results indicate that this was a rather justifiable procedure for the practical case studied. An efficient procedure for eliminating poor work and recreation schedule candidates, however, would definitely increase the possibility to solve the ILP-problems within reasonable limits of computer time and memory requirements. This seems to be an interesting direction for further research. Also further research is

Nodes		1	2	3	4	5	6	7	8	9	10
	Schedule types	20	20	50	50	62	62	62	62	62	62
1	20	-	20	30	30	5	5	5	5	5	5
2	20	20	-	30	30	5	5	5	5	5	5
3	50	4	4	-	1	17	17	17	17	17	17
4	50	4	4	1	-	17	17	17	17	17	17
5	62	18	18	3	3	-	2	2	2	2	2
6	62	18	18	3	3	2	-	2	2	2	2
7	62	18	18	3	3	2	2	-	2	2	2
8	62	18	18	3	3	2	2	2	-	2	2
9	62	18	18	3	3	2	2	2	2	-	2
10	62	18	18	3	3	2	2	2	2	2	-

Figure 11 : Aversion matrix for B747 maintenance with CS = 7, SL = 90 %
and crew splitting allowed.



Start Node	Total aversion	Shift schedule
1 ▲	68	$x_{50}\ x_{20}\ x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{20}\ x_{50}$
2 ▲	43	$x_{20}\ x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{50}\ x_{50}\ x_{20}$
3 ▲	43	$x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{62}\ x_{50}\ x_{50}\ x_{20}\ x_{20}$

Figure 12 : Shift schedules for B 747 maintenance with CS = 7, SL = 90 % and crew splitting allowed.

needed to exploit the particular structure of the average aversion matrix in order to develop optimal procedures for solving the traveling salesman version of the work and recreation schedule sequencing problem.

REFERENCES

1. BUYSSE, W.R., Air Transport Carriers Operating Fixed Flight Schedules - Mainbase aircraft line and light maintenance manpower allocation and scheduling - Volume I and II, MBA Thesis, Department of Applied Economic Sciences, KU. Leuven, 1985.

2. CHOW, G.C., Test of equality between sets of coefficients in two linear regressions, Econometrica, 28(3), 1960.

3. KIEFER, J., K-sample analogue of the Kolmogorov-Smirnov and Cramer-Von Mises tests, The Annals of Mathematical Statistics, 30, 1959, 420-447.

4. PURI, M.L., SEN P.K., Nonparametric methods in multivariate analysis, John Wiley & Sons, Inc., 1971.

5. RANDLES, H.R., WOLFE, D.A., Introduction to the theory of nonparametric statistics, John Wiley & Sons, Inc., 1979.

6. STERN, H.I., HERSH, M., Scheduling aircraft cleaning crews, Transportation Science, 14(3), August 1980, 277-291.

7. TIEN, J.M., KAMIYAMA, A., An manpower scheduling algorithms, SIAM Review, 24(3), July 1982, 275-287.

8. WARNER, M.D., Scheduling nursing personnel according to nursing preference : A mathematical programming approach, Operations Research, 24(5), 1976, 842-856.